Errata: Analysis and Approximation of Rare Events Representations and Weak Convergence Methods,
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Amarjit Budhiraja and Paul Dupuis

Corrections as of 5 January, 2021

1. **page 459.** It was pointed out to us by Frédéric Cérou and Mathias Rousset that there is an error in Definition 16.12. This error does not appear in reference [77], which is the source for the material of this chapter, but was introduced when attempting to rewrite the definition used there to directly incorporate boundary and/or terminal conditions. This was driven by a desire to use a single characterization of subsolutions in Chapter 17 that could cover both the importance sampling and splitting frameworks. The corrected definition is as follows.

   Definition 16.12 *A continuous function $\bar{V}:\mathbb{R}^d \to \mathbb{R}$ is a subsolution if it is bounded from below,*
   $$\bar{V}(y) \leq \inf_{\phi \in K_{y,T}, 0 \leq t \leq T < \infty} \left[\int_0^t L(\phi(s), \dot{\phi}(s)) ds + \bar{V}(\phi(t)) \right]$$
   for all $y \in (A \cup B^\circ)^c$, and $\bar{V}(z) \leq 0$ for $z \in B$.

   The problem with the original definition is that without the additional infimization on t the definition as stated does not reduce to the differential form used in the chapters on importance sampling (see e.g. Definition 14.4 for a classical subsolution). The only result in the chapter which requires the change is Theorem 16.18, whose proof is incorrect unless the revised definition is used, since the T appearing at the top of page 466 in the expression
   $$\left[\int_0^T L(\phi(s), \dot{\phi}(s)) ds - (\bar{V}(x_0) - \bar{V}(\phi(T))) \right]$$
   need not be such that $\phi(T) \in B$, and instead we only know that $\phi(T) \in (A \cup B^\circ)^c \cup B$. More precisely, the right side of the inequality in the top display of page 466 should read as
   $$\inf_{\phi \in K_{x_0,T}, 0 \leq t \leq T < \infty} \left[\int_0^t L(\phi(s), \dot{\phi}(s)) ds - (\bar{V}(x_0) - \bar{V}(\phi(t))) \right].$$
   With the corrected definition of a subsolution this quantity is always non-negative, as required in the proof.

   The error propagates through the chapter, and here we list the places where an analogous change is required. Remark 16.13 is still valid, though in the last display on page 459 the two T's on the right hand side should be t with $t \in [0, T]$. The display appearing in Remark 16.19 should read
   $$\bar{V}(y) \leq \inf_{\phi \in K_{y,T}, 0 \leq t \leq T < \infty} \left[\int_0^t [L(\phi(s), \dot{\phi}(s)) - \varepsilon] ds + \bar{V}(\phi(t)) \right].$$
   Definitions 16.20 and 16.21 should be modified in the analogous way to read as follows.

Definition 16.20 *A continuous function* $\bar{V} : \mathbb{R}^d \times [0,T] \to \mathbb{R}$ *is a subsolution if it is bounded from below,*

$$\bar{V}(y,t) \leq \inf_{\phi \in \bar{K}_{y,t,T}} \left[\int_t^\tau L(\phi(s), \dot{\phi}(s))ds + \bar{V}(\phi(\tau), \tau) \right]$$

for all $(y,t) \in \mathbb{R}^d \times [0,T)$, $\tau \in [t,T]$, *and* $\bar{V}(z,T) \leq 0$ *for* $z \in B$.

Definition 16.21 *A continuous function* $\bar{V} : \mathbb{R}^d \times [0,T] \to \mathbb{R}$ *is a subsolution if it is bounded from below,*

$$\bar{V}(y,t) \leq \inf_{\phi \in \bar{K}_{y,t,\sigma}, t \leq \tau \leq \sigma \leq T} \left[\int_t^\tau L(\phi(s), \dot{\phi}(s))ds + \bar{V}(\phi(\tau), \tau) \right]$$

for all $(y,t) \in \mathbb{R}^d \times [0,T)$ *and* $\bar{V}(z,t) \leq 0$ *for* $z \in B$ *and* $t \in [0,T]$.

Finally, the bottom of page 468 should read as follows: The definition of $W(y,z)$ then gives [for all $\phi \in \bar{K}_{y,t,T}, t \leq \tau \leq T$ with $z = \phi(\tau)$] that

$$\bar{V}(y) \leq \bar{V}(\phi(\tau)) + \int_t^\tau L(\phi(s), \dot{\phi}(s))ds.$$

2. **page 468.** In the last display on this page $\int_0^T$ should be $\int_t^T$.